Passive Magnetic Momentum Wheel Unloading

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Rotational kinetic energy may be extracted from the momentum wheels of a spacecraft by converting the rotational energy to electrical energy with a dynamo effect using the Earth's magnetic field, and dissipating the resultant kinetic energy as heat. This is an extension of the magnetic spin and nutation damping schemes used with spinning spacecraft. A three-axis momentum wheel assembly can be constructed with the desired inertial and autounloading characteristics that will operate in low Earth orbit. For synchronous altitudes, such a system is also possible, but the wheel design is affected greatly. The passive system is essentially failure-proof and results in a simplification of onboard systems and ground operations.

Nomenclature

B = flux density, G
B_m = peak flux density, G
d = diameter, cm
dr = infinitesimal of length, cm
E = energy, ergs
H = magnetic field intensity, Oe
H_m = peak magnetic field intensity, Oe
I = current, A
l = length, cm
p = power, erg/sec
r = radius, cm
R = resistance, Ω
t = time, sec
T = torque, dyn-cm
T_e = eddy current torque, dyn-cm
T_h = hysteresis torque, dyn-cm
T_{sc} = sheathed coil torque, dyn-cm
T_{total} = total resultant torque
V = volume, cm^3
v = voltage, V
ϕ = flux, Mx
ρ = resistivity, Ω-cm
ω = spin rate, rad/sec
$<\oint H dB>$ = average traversed area of hysteresis loop

Introduction

THE basic methods of using magnetic materials to interact with the Earth's field to reduce spin and angular motions of spacecraft are well understood and developed.[1] The Vanguard spacecraft launched in 1958 was noted to have its spin rate reduced due to eddy current losses in the aluminum skin.

The technology of using highly permeable rods in the spacecraft structure to reduce spin rate and nutation, and to achieve magnetic orientation for subsequent gravity gradient capture, was developed at the Applied Physics Laboratory of Johns Hopkins University. The physical and mathematic basis of this scheme is developed in Ref. 1. This paper presents an extension of that technology to momentum dumping of reaction wheel assemblies.

In general, reaction wheels are used in two ways aboard spacecraft. In one scheme, a wheel is kept spinning at a fairly constant rotational rate to establish a preferred momentum vector. An active control system maintains the wheel speed. It will become obvious that the momentum unloading scheme discussed here is undesirable in this case and for any other biased momentum system.

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A more interesting case involves three orthogonal reaction wheels in a zero-momentum system. Ideally, the wheel speeds remain zero unless it is desired to maneuver the vehicle. In that case, the control system would spin one or more wheels in one direction, and because of conservation of momentum, the vehicle would move in the opposite direction at a rate determined by the ratio of the inertia of the vehicle in that axis to the inertia of the wheel. At the end of the maneuver, the wheel speed would be reduced to zero by the control system, and the vehicle would stop. Thus, the system state is nominally zero momentum. This can be the case, however, only in the absense of external torques. External torques are present, although small, and result from gravity-gradient, solar pressure, and atmospheric drag, among other effects. Thus, there is a net secular increase in reaction wheel speed, representing a net increase in system momentum due to the effects of external torques.

This momentum must be "dumped," or else the wheel's synchronous speed soon would be exceeded, and the wheel would be useless for maneuvering. Typical spacecraft require momentum unloading every few hours or days. This is accomplished by supplying a controlled external torque on the vehicles by reaction jet assemblies, or by pushing against the Earth's magnetic field by means of large coils that can be energized from onboard power sources.

Both schemes commonly are used, but neither is desirable: in the first case because of the expendable, and in the latter because of the use of spacecraft electrical power. The wheel speed must be monitored, trends plotted, and the timing and details of the unload performed. In general, spacecraft momentum maintenance is a burden for onboard systems and ground-based operations.

This paper discusses the concept of a passive wheel unloading mechanism that provides what may be thought of as a reactionless friction that serves to slow the wheel down without the corresponding reaction on the spacecraft. This method of momentum unloading can result in a weight savings and increased reliability in the onboard system.

The method used to achieve these results is a direct extension of the methods described in Ref. 1 for spinning spacecraft. In general, the concept is to include a highly permeable material in the structure of the wheel itself, so that a fraction of the rotational energy is converted to electrical energy by the rotation of the material in the Earth's magnetic field. Then the electrical energy is dissipated as heat. Note that dissipation mechanism always works so as to reduce wheel speed. Thus, it tends also to impede wheel speed increases by the control system, but this effect can be shown to be so small as to be negligible.

After a discussion of the physics of the situation, an example will be discussed using this passive unloading system with two orthogonal rods for a typical low Earth orbiting spacecraft. The situation for a synchronous spacecraft will be touched upon. The author hopes to include this effect in a three-axis spacecraft simulation to study the dynamics of the situation further over many orbit periods.

Energy Damping Mechanism

Three mechanisms for energy removal will be discussed here: eddy current, hysteresis, and sheathed coil damping. For further discussion and the derivations, the interested reader is referred to Ref. 1.

Eddy Current Damping

Consider the case of a long, slender permeable rod rotating in a magnetic field (Fig. 1). For simplicity, the rod cross section is circular. The induced voltage about a cylindrical element due to the flux density in the rod can be found easily.[1] From the inherent resistance in the rod, an average power dissipation per revolution may be defined:

$$P = (-\pi\omega^2 r^3 B_m^2 l dr \times 10^{-9})/4p \text{ ergs/sec}$$

For a system of two orthogonal rods, we may consider the separation effect coefficient[1] to be one and derive the resultant torque on the system from the energy loss terms. For this term only, the spin rate ω may be shown to decrease exponentially in time. The T_e term represents a torque resulting from eddy current effects which acts in opposition to the rotation and is proportional to the rotational rate, the rod length, the flux density in the rod (and hence the magnetic field strength H), and the fourth power of the radius of the rod, while being inversely proportional to the rod resistivity. If we re-express this term as a function of the slenderness parameter l/d, we have

$$T_e(\omega) = (-B_m^2/8p)(l/d)r^5\omega \times 10^{-9} \quad \text{dyn-cm}$$

Thus, the eddy-current-derived torque is strongly dependent on the rod radius and is proportional to rotation rate. For example, for a rod of the material AEM4750, having $p = 51.2 \times 10^{-6}$ Ω-cm, $l/d = 124$, $r = 0.1$ cm, $B_m = 10^3$ G, at $\omega = 100$ rad/sec, $T_e \cong 0.3$ dyn-cm.

Hysteresis Damping

Hysteresis losses may be viewed as the conversion of magnetic motion to heat by the internal friction of magnetic domains in the permeable material. These losses are not a function of the spin rate ω, and so theoretically the spin rate may be reduced to zero with this mechanism. For a system of two orthogonal rods, the net resultant torque per revolution may be shown to be

$$T_h = \frac{-\langle \oint H dB \rangle}{2\pi} \frac{(l)}{(d)} r^3 \text{ dyn-cm}$$

This torque depends on the l/d parameter as before and is a strong function of r. For low wheel speeds, the magnitude of this torque will be larger in magnitude than the eddy current term and will dominate. For a rod of AEM4750, $l/d = 124$, $r = 0.1$ cm, and $\langle \oint H dB \rangle = 120$ G-Oe, $T_h = -2.368$ dyn-cm.

Sheathed Coil Damping

In this section, the equations for the removal of rotational energy by sheathed coil damping will be developed. Sheathed coil damping will be shown to be a special case of shorted coil damping, as developed in Ref. 1.

If the permeable rod in Fig. 1 is wound with many turns of wire and the wire is connected to form a closed loop, the principle of shorted coil damping may be illustrated. As the rod spins in the Earth's magnetic field, it induces a voltage in the coil which is dissipated by I^2r losses. The effect is proportional to the number of turns in the coil squared, but an interesting case is the seemingly degenerate one of a "single-turn" coil.

Consider the hollow cylinder whose inside diameter matches the outside diameter of a permeable rod. The rod is contained within the conducting cylinder, as shown in Fig. 2. By an analogy to the case of "skin effect" in conduction of high-frequency currents, the rod should be inset from the end of the cylinder by at least its own diameter. In Fig. 2, $l_i = d_i$. Plugs of the conductor then would be inserted in the ends, so that the rod would be enclosed completely in the conductor. The voltage around a cylindrical element in the bar is given by Faraday's law:

$$V = -\frac{d\phi}{dt} \times 10^{-8} \text{ V}$$

where

$$\frac{d\phi}{dt} = \pi r^2 B_m \omega \cos\omega t \quad \text{Mx/sec}$$

The resistance of the shell (Fig. 3) is given by

$$R = 2\pi pr/ldr \ \Omega$$

Then the power dissipated is given by

$$P = \frac{V^2}{R} = \frac{-\pi r^3 \omega^2 B_m^2 \cos^2\omega t \times 10^{-16} ldr}{2p} \text{ W}$$

Now, for the total power we integrate over all of the cylindrical elements, from the outside radius of the permeable rod to the outside radius of the conductor:

$$P_T = P(r)dr = -\frac{-\pi\omega^2 B_m^2 \cos^2\omega t \times 10^{-16} l}{2p} \int_{d1/2}^{d_2/2} r^3 dr \text{ W}$$

Over 1 rev, the average of $\cos^2\omega t = ½$. The units will be converted to ergs per second by (1 erg/sec = 10^{-7} W)

$$P_T = (-\pi\omega^2 B_m^2 10^{-9} l/16p)(r_2^4 - r_1^4) \text{ ergs/sec}$$

Similar to the previous cases, we define the average torque for a revolution as

$$T_{sc} = \frac{1}{2\pi\omega} P_t = \frac{-B_m^2 l \times 10^{-9}}{32p}(r_2^4 - r_1^4)\omega \text{ dyn-cm}$$

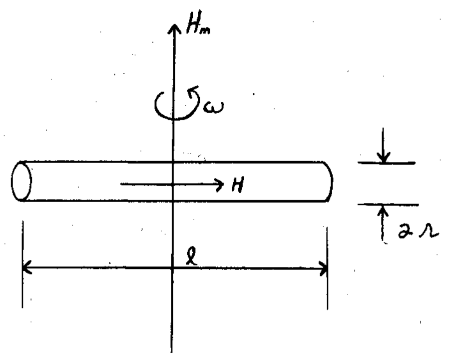

Fig. 1 Permeable rod rotating in a magnetic field.

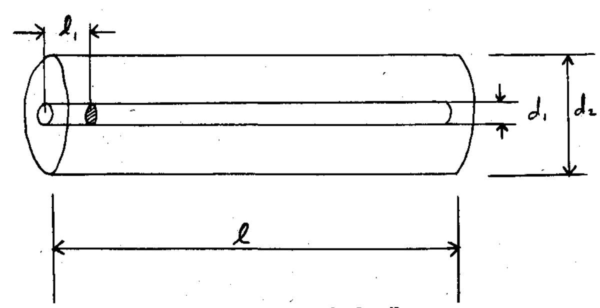

Fig. 2 Sheathed coil.

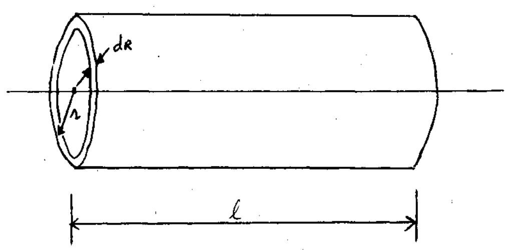

Fig. 3 Conducting shell.

The heavy dependence on r is seen again. If we consider that the outside diameter is much larger than the inside diameter, $r_2 > r_1$ such that $r_2^4 >> r_1^4$, then

$$T_{sc} \cong \frac{-B_m^2 \times 10^{-9}}{16p} \left(\frac{l}{d}\right) r^5 \omega \text{ dyn-cm}$$

The expression for the system of two orthogonal rods will be

$$T_{sc} = \frac{-B_m^2 \times 10^{-9}}{8p} \left(\frac{l}{d}\right) r^5 \omega \text{ dyn-cm}$$

Note that this expression has the same form as that derived for eddy current damping, as the losses in this case are eddy current losses in the aluminum sheathing, whereas in the previous case only eddy current losses in the magnetic rod itself were considered.

Here, the l/d parameter refers to the sheathing material and not to the rod, as before. For a single rod of AEM4750, sheathed in aluminum of $p = 2.6 \times 10^{-6}\Omega$-cm, with $l/d = 25$, $B_m = 10^3$ G, for $r = 1$ cm, at $\omega = 100$ rad/sec, $T_{sc} = 1.2 \times 10^5$ dyn-cm. An aluminum cylinder of 1 cm diam, with the requirement that the magnetic rod diameter is much less that this, allows a sheath l/d of around 25, with a rod diameter of 0.1, and $l/d = 124$, as before.

Defining a structure consisting of two orthogonal sheathed magnetic rods, with rod material = AEM4750, $l/d = 124$, $r = 0.1$ cm, $l = 248$, sheathing material = aluminum, o.d. = 1 cm, $l/d = 24.8$, $\omega = 100$ rad/sec, $T_e = -3.027 \times 10^{-1}$ dyn-cm, $T_h = -2.368$ dyn-cm, $T_{sc} = -1.2 \times 10^5$ dyn-cm, $T_{\text{total}} \cong -1.2 \times 10^5$ dyn-cm ($\cong 10^{-2}$ ft-lb, $\cong 2$ oz – in.).

Parametric Curves

The hysteresis loop for AEM4750 is given in Ref. 1, along with curves depicting the effect of l/d ratio on the loop, the effects of the magnitude of H on loop area and peak flux density, and separation parameter effects.

Wheel Design for Self-Unloading

An inertia wheel assembly may be modeled as a two-spoked wheel with hub, driven by a motor via a shaft (Fig. 4). The spokes would be the sheathed magnetic rods, and the hub would be aluminum to supply the required inertia. The hub will contribute the largest inertia factor. The inner radius of the hub is set by the rod length, and the wall height is determined by the rod diameter and must equal at least twice the diameter, since the rods must pass without intersection.

For a typical low Earth orbiting spacecraft of the Nimbus-G type, the required wheel inertia would be 10^5 g-cm^2 (10^{-2} slug-ft^2), as in the Nimbus-G pitch axis. Choosing $h = 2$ cm, $r_{\text{out}} = 13$ cm, and with an orthogonal set of two 0.1-cm rods as discussed before, then the total inertia of the hub and spoke assembly will be 1.47×10^5 g-cm^2. The overall diameter would be 26 cm, height 2 cm, and weight 322 g, exclusive of drive motor and case. The actual Nimbus-G pitch wheel assembly, including drive motor, case, and tach, weighs about 4 kg. The magnetic unloading torque on the wheel would have a maximum value of $-(2.368 + 1.2 \times 10^3 \omega)$ dyn-cm (ω in rad/sec).

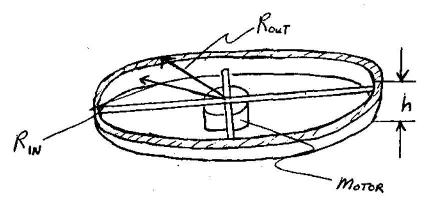

Fig. 4 Example of inertia wheel assembly.

Expected Performance at Low Altitude

For a typical low Earth orbiting spacecraft of the Nimbus-G type, with a Polar orbit at around 1000-km alt, the peak gravity gradient torque is 10^3 dyn-cm (10^{-4} ft-lb) and represents the largest external torque on the vehicle. The passive magnetic unloading torque for the wheel discussed here is $1.2 \times 10^3 \omega$ dyn-cm, so that the wheel indeed could handle the disturbance environment in this application and should be able to unload the wheel completely in most cases. However, because of momentum cross-coupling, the dynamics of a three-axis control system using passive magnetic unloading would be done best by simulation. The author is planning to incorporate these features in a Nimbus-G simulation that presently is being developed. (Note: the Nimbus design uses a biased wheel pair in the roll axis. This scheme could not be used here.)

Expected Performance at Synchronous Altitude

At synchronous altitude, the passive wheel unloading system is less effective, because of the rapid fall-off of the Earth's magnetic field strength. Balancing this are the lower external disturbances encountered. The gravity gradient, magnetic, and aerodynamic torques on a spacecraft fall off drastically, and the predominant disturbance term is the solar torque. On the ATS-6 vehicle at synchronous altitude, the peak disturbance torque is seen in the pitch axis and amounts to 10^2 dyn-cm (10^{-5} ft-lb). For applications at synchronous altitude, physically larger wheels, i.e., with longer rods, could be used, with adjustments in the hub mass to achieve the desired inertia.

For the ATS-6 spacecraft, the solar torque value is rather high, because of the large spacecraft size and the large-area solar panels on considerable lever arms. However, this case will be used for discussion. The ATS-6 reaction wheel assembly provides approximately 10^6 g-cm^2 (0.0647 slug-ft^2) of inertia. The magnetic field strength value has fallen to 0.001 Oe at synchronous altitudes, which, for the magnetic materials available, give a flux density on the order of 1.0 G. As in the low-altitude case, the hysteresis losses and eddy current effect will be small with respect to the sheathed coil case. The geometry of the wheel will change, because of the unloading torque dependence on l/d and r^n terms.

For an order-of-magnitude discussion, we could choose a target-unloading torque of 50 dyn-cm (half of peak) at 100 rad/sec. Using

$$T_{sc} = \frac{-B_m^2 \times 10^{-9}}{8p} \frac{l}{d} r^5 \omega \text{ dyn-cm}$$

with $T_{sc} = -50$ dyn-cm, $B_m = 1$ G, $p = 2.6 \times 10^{-6}$ Ω-cm, $\omega = 100$ rad/sec, and l/d and r^5 as variable parameters, for the aluminum sheath, then

$$\frac{1 \times 10^4}{(l/d)} \cong r^5$$

Choosing $l/d = 10$, then $r \cong 4.0$, and $l = 80$ cm, the wheel assembly will be approximately 0.8 m diam. Keeping the same l/d ratio for the rods (124), the rod diameter will be 0.37 cm and thus more effective. The inertia of this configuration can be calculated:

$$I_{\text{rods}} = d\pi r^2 l(3r^2 + l^2) = 7.0 \times 10^7 \text{ g-cm}^2$$

Where $d = 2.7$ g/cm^3 (value for aluminum used). Thus, the required inertia is exceeded by the rod structure alone in this simple case. For synchronous altitude, the passive magnetic scheme could be incorporated in existing wheel designs to aid the jet unloading system, rather than attempt to design a totally passive system with a radical wheel design. The passive system also would provide some unloading capability in the case of failure of the active system.

Advantages of Self-Unloading Reaction Wheel Assemblies

For low Earth orbiting satellites, the passive magnetic system described here could handle all of the required momentum balance. At synchronous altitudes, at least part of the momentum unloading could be done passively, without greatly affecting the wheel design.

The advantages of this system lie not only in the reduction of reaction fuel budgeted for unloading or the deletion of an active unloading system, but also in the simplification of the ground operations task of momentum management. For example, on the OAO spacecraft (Copernicus), the active magnetic unloading system weighs about 17 kg and uses about 10 W of electrical power. The original expendables budget included enough gas for 1 yr of jet unloading.

The ATS-6 spacecraft budgeted about 3 kg of hydrazine for unloading for 3 yr of operation at synchronous altitude. In both of these active systems, ground support is required to calculate the unloading times and to command the jets or magnetic torquers. (The Copernicus magnetic torquers remain on almost continuously.) A failure in any part of the system would allow momentum buildup and consequent uselessness of the system. The beauty of a passive system is that it is essentially failure-proof if it was designed to operate properly at the beginning.

None of the calculations for optimum unloading times need be done, as the wheel assembly will stay around zero momentum in steady state. The unloading operation proceeds continuously, without disturbance to the vehicle altitude. Currently used unloading systems (jets, or active magnetic) provide an external torque to the vehicle which results in a small attitude change that imposes a wheel motion toward zero revolutions. The passive system described here results in no external torque on the vehicle and, thus, no attitude change. Passive momentum unloading can be included in reaction wheel design for future low Earth orbiting missions, to augment or replace active momentum unloading systems, and to simplify mission planning and operations.

Acknowledgment

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References

[1] Fischell, R.E., "Magnetic Damping of the Angular Motions of Earth Satellites," *ARS Journal*, Vol. 31, Sept. 1961, pp. 1210-1217.